## Entanglement entropy of nonequilibrium steady states

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The entanglement entropy has been attracting much attention recently in various fields of physics, because it is found useful not only for quantifying the amount of resources for quantum information tasks,but also for analyzing physical properties such as quantum phase transitions. For concreteness, consider a one-dimensional quantum system, whose length is very large, posssibly infinite. For its pure state, the entanglement entropy is defined as the von Neumann entropy $S_L$ of the reduced density operator, obtained from the pure state of the total system, of a subsystem of finite length $L$. Of particular interest is the size scaling, i.e., the $L$ dependence of $S_L$, on which numerous works have been reported. Most previous works studied $S_L$ of energy eigenstates. According to thermodynamics and the eigenstate thermalization hypothesis, it is expected that $S_L = O(L)$ for eigenstates of energy $O(L)$ whereas $S_L = o(L)$ for eigenstates of energy $o(L)$ including the ground state. The former behavor is universal (under the eigenstate thermalization hypothesis) and therefore has no information on the system because it just says that the entropy is extensive. By contrast, more detailed behaviors of the latter reflect important properties of the system.

When the ground state is unique and separated from excited states by a finite gap, the *area law* $S_L = O(1)$ is obtained [1] under the physically reasonable condition that the Hamiltonian is the sum of local interactions that are bounded. For gapless systems, where the ground state is not separated by a finite gap, $S_L$ can be larger. For example, the *logarithmic law* $S_L = O(\ln L)$ is found in many systems [2-4], including the system of free fermions [5]. Larger $S_L$, such as $S_L = O(L^{1/2})$, is found in some systems with internal degrees of freedom (such as a large spin) [6]. For systems *without* translational invariance, even larger $S_L$ is possible, such as the *volume law* $S_L = O(L)$ [7,8]. The $L$ dependence of $S_L$ of the ground states, i.e. equilibrium states at zero temperature, have thus been studied extensively. By contrast, $S_L$ of nonequilibrium steady states (NESSs) remains almost unexplored.

In this talk, we discuss the size scaling of the entanglement entropy $S_L$ in the NESS of a one-dimensional noninteracting fermionic system in a random potential. It models a mesoscopic conductor, composed of a long quantum wire, with a random potential, and two electron reservoirs. The difference $\Delta\mu$ of the chemical potentials of the reservoirs induces a steady current, and a NESS is realized in the quantum wire. We assume zero temperature for the reservoirs, and consequently the total system is in a pure quantum state. The $S_L$ is defined as the von Neumann entropy of a subsystem of length $L$ in the quantum wire. At equilibrium ($\Delta\mu = 0$), $S_L$ obeys the logarithmic law $S_L = O(\ln L)$. In NESSs ($\Delta\mu > 0$), however, $S_L$ grows anomalously fast obeying the 'quasi volume law,' $S_L = L\eta(L)|\Delta k_F| + O(\ln L)$. Here, $\eta(L)$ is a positive function gradually decreasing with increasing $L$, and $\Delta k_F$ is the difference of the Fermi wavenumbers of the reservoirs.

This anomalous behavior arises from *both* nonequilibrium and multiple scatterings by the random potential, which change drastically the correlations of *all* local observables up to two-site operators.

More details will be presented at Hakoshima's talk [9].

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